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# $\mathrm{B_s} - \overline{\mathrm{B}}_\mathrm{s}$ mixing in an SO(10) SUSY GUT model*

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**Abstract.** We perform a renormalisation group analysis of the SO(10) model proposed by Chang, Masiero and Murayama, which links the large atmospheric neutrino mixing angle to loop-induced transitions between right-handed $b$ and $s$ quarks. We compute the impact on $\mathrm{B_s} - \overline{\mathrm{B}}_\mathrm{s}$ mixing and find that the mass difference in the $\mathrm{B_s}$ system can exceed its Standard Model value by a factor of 16.

**PACS.** 12.60.Jv supersymmetric models – 12.15.Ff Quark and lepton masses and mixing

## 1 Introduction

Grand Unified Theories combine quarks and leptons into symmetry multiplets. This opens the possibility to find links between the flavour structures of both sectors. Chang, Masiero and Murayama (CMM) have proposed an interesting supersymmetric SO(10) model in which the large $\nu_\mu$–$\nu_\tau$ mixing angle can affect transitions between right-handed $b$ and $s$ quarks through supersymmetric loop diagrams [1]. The SO(10)-symmetric superpotential has the form

$$W_{10} = \frac{1}{2}\mathbf{16}^T Y^U \mathbf{16}\,\mathbf{10_H} \; + \; \frac{1}{M_{\mathrm{Pl}}}\frac{1}{2}\mathbf{16}^T \tilde{Y}^D \mathbf{16}\,\mathbf{10'_H}\mathbf{45_H} + \frac{1}{M_{\mathrm{Pl}}}\frac{1}{2}\mathbf{16}^T Y^M \mathbf{16}\,\overline{\mathbf{16}}_\mathbf{H}\overline{\mathbf{16}}_\mathbf{H}. \qquad (1)$$

Here $\mathbf{16}$ is the usual spinor comprising the matter superfields and the other fields are Higgs superfields in the indicated representations. The Yukawa coupling $Y^U$ is a symmetric $3\times 3$ matrix in generation space containing the large top Yukawa coupling. The two dimension-5 terms involve the Planck mass $M_{\mathrm{Pl}}$ and further Higgs fields in the indicated representations. The last term in (1) generates small neutrino masses via the standard see-saw mechanism, once $\overline{\mathbf{16}}_\mathbf{H}$ acquires an SO(10)-breaking VEV. From $m_t \gg m_c$ we observe a large hierarchy in $Y^U$, which must be largely compensated in $Y^M$ in order to explain the observed pattern of neutrino masses. This is achieved in a natural way by invoking flavour symmetries, the simplest of which render $Y^U$ and $Y^M$ simultaneously diagonal. In this basis the remaining Yukawa matrix $\tilde{Y}^D$ has the form

$$\tilde{Y}^D = V^*_{\mathrm{CKM}} \begin{pmatrix} \tilde{y}_d & 0 & 0 \\ 0 & \tilde{y}_s & 0 \\ 0 & 0 & \tilde{y}_b \end{pmatrix} U_{\mathrm{PMNS}}. \qquad (2)$$

Here $V_{\mathrm{CKM}}$ and $U_{\mathrm{PMNS}}$ are the CKM and PMNS matrices encoding flavour mixing in the quark and lepton sectors [2] and certain diagonal phase matrices have been omitted in (2). From (2) one realizes that the flavour structure of $\tilde{Y}^D$ is neither symmetric nor anti-symmetric. This is possible, because the corresponding dimension-5 term in (1) transforms reducibly under SO(10). At some scale $M_{10}$ between the Planck and GUT scales the $\mathbf{45_H}$ and $\overline{\mathbf{16}}_\mathbf{H}$ acquire VEVs and SO(10) is broken to SU(5). The SU(5) superpotential reads

$$W_5 = \frac{1}{2}\Psi^T Y^U \Psi \mathbf{5_H} + \Psi^T Y^D \Phi\, \overline{\mathbf{5}}_\mathbf{H} + \Phi^T Y^\nu N\, \mathbf{5_H} + \frac{1}{2}\frac{v^2_{\overline{16}}}{M_{\mathrm{Pl}}} N^T Y^M N. \qquad (3)$$

The matter supermultiplets $\Psi$, $\Phi$ and $N$ are the usual $\mathbf{10}$, $\overline{\mathbf{5}}$ and $\mathbf{1}$ from the decomposition of the $\mathbf{16}$. Furthermore, $Y^D \propto \tilde{Y}^D v_{45}/M_{\mathrm{Pl}}$ and $v_{45}$ and $v_{\overline{16}}$ are the VEVs of the $\mathbf{45_H}$ and $\overline{\mathbf{16}}_\mathbf{H}$ fields. The SO(10) Higgs fields comprise the SU(5) ones as $\mathbf{10_H} \supset \mathbf{5_H}$ and $\mathbf{10'_H} \supset \overline{\mathbf{5}}_\mathbf{H}$. The remaining components of the SO(10) Higgs fields are assumed heavy with zero VEVs. Finally, at the GUT scale the SU(5) theory is broken to the MSSM and the MSSM Higgs fields $H_u \subset \mathbf{5_H}$ and $H_d \subset \overline{\mathbf{5}}_\mathbf{H}$ couple to up- and down-type fermions, respectively.

The soft SUSY-breaking terms are assumed universal near the Planck scale. The large Yukawa coupling in $Y^U$ now renormalises the squark mass matrix. The renormalisation group (RG) flow down to $M_{10}$ (and to $M_{\mathrm{GUT}}$)

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will keep its diagonal form (in the basis in which $Y^U$ and $Y^M$ are diagonal), but will split the mass of the third from those of the first two generations. The diagonalisation of $Y^D$ involves the rotation of $\Phi$ in (3) with $U_{\mathrm{PMNS}}$. Since $\Phi$ unifies the left-handed (s)leptons with righthanded (s)quarks, the large atmospheric neutrino mixing angle will appear in the mixing of right-handed $\tilde{b}$ and $\tilde{s}$ squarks.

We have performed a complete RG analysis of the CMM model and computed its impact on $\mathrm{B_s}-\overline{\mathrm{B}}_\mathrm{s}$ mixing. The key parameter in the analysis is the top Yukawa coupling $y_t$, which drives the $\tilde{b}_R$–$\tilde{s}_R$ mixing effect. The fact that the RG evolution of this coupling is governed by infrared quasi-fixed points in SO(10), SU(5) and the MSSM allows us to place an upper bound on $\mathrm{B_s}-\overline{\mathrm{B}}_\mathrm{s}$ mixing. At this point we remark that our result corresponds to the Higgs sector specified above (supplemented by an additional **16** to avoid unwanted D-term breaking), which is minimal in its effect on the RG evolution of $y_t$ and leads to the weakest possible bound on $\mathrm{B_s}-\overline{\mathrm{B}}_\mathrm{s}$ mixing. Changing e.g. the last term in (1) into a dimension-4 coupling involving a $\overline{\mathbf{126}}_\mathbf{H}$ Higgs field while keeping the low-energy parameters $m_t$ and $\tan\beta$ fixed will strengthen our bound on $\mathrm{B_s}-\overline{\mathrm{B}}_\mathrm{s}$ mixing further.

## 2 The top Yukawa coupling

The large top Yukawa coupling $y_t$ suppresses the third-generation squark (and slepton) masses through its RG effects and generates the desired flavour-changing neutral $\tilde{b}_R$–$\tilde{s}_R$ transitions. In both SO(10) and SU(5) $y_t$ possesses an infrared quasi-fixed point corresponding to a fixed point of the ratio $y_t/g$, where $g$ is the gauge coupling. From the values of $m_t$ and $\tan\beta$, which is the ratio of the VEVs of $H_u$ and $H_d$, one can compute $y_t \propto m_t/\sin\beta$ at the electroweak scale and evolve it up to $M_{\mathrm{GUT}}$, $M_{10}$ and the fundamental scale near $M_{\mathrm{Pl}}$. The running above $M_{10}$ is much stronger than between $M_{\mathrm{GUT}}$ and $M_{10}$ because the group-theoretical factors in SO(10) are larger than in SU(5). Next we consider the critical Yukawa coupling $y_t^c$, which corresponds to the quasi-fixed point in SO(10), i.e. $y_t^c/g$ is constant above $M_{10}$. $y_t^c$ is shown as a dashed curve in Fig. 1, with the two vertical lines indicating the scales $M_{\mathrm{GUT}}$ and $M_{10}$. For $y_t < y_t^c$ one consequently finds $y_t$ small at high energies. The solid line in Fig. 1 illustrates this situation for the following input parameters:

| $m_t^{\mathrm{pole}}$ | $\alpha_s(M_Z)$ | $\alpha_2(M_Z)$ | $\alpha_1(M_Z)$ | $\tan\beta$ |
|---|---|---|---|---|
| 174 GeV | 0.121 | 0.034 | 0.017 | 3 |
| $m_{\tilde{q}}$ | $m_{\tilde{t}_1}$ | $m_{\tilde{t}_2}$ | $\theta_{\tilde{t}}$ | $m_{\tilde{g}}$ |
| 300 GeV | 200 GeV | 300 GeV | $\pi/6$ | 400 GeV |

We have computed all relevant RG coefficients in SO(10) and SU(5) using the general result of [3]. For the RGE's above $M_{\mathrm{GUT}}$ we work in the leading logarithmic approximation, but include next-to-leading-order corrections in the MSSM. In this way we can account for electroweak threshold corrections, which become relevant if $y_t$ is close to $y_t^c$. For instance, $y_t$ depends on the listed squark masses through these corrections. $\alpha_{1,2,s}$ are the usual squared MSSM gauge couplings divided by $4\pi$, $\theta_{\tilde{t}}$ is the stop mixing angle and $m_{\tilde{g}}$ is the gluino mass. One finds $y_t$ raised to $y_t^c$ for $(m_t, \tan\beta) = (180\,\mathrm{GeV}, 3)$ or for $(m_t, \tan\beta) = (184\,\mathrm{GeV}, 4)$ with the remaining parameters unchanged. For very small $\tan\beta < 2$, the fixed-point values can be exceeded and the coupling typically becomes nonperturbative below the Planck scale. In this case the model loses its predictivity. Since further such low values of $\tan\beta$ are strongly disfavoured by LEP data, we require $y_t \le y_t^c$.

## 3 $\mathrm{B_s}-\overline{\mathrm{B}}_\mathrm{s}$ mixing

The $\mathrm{B_s}-\overline{\mathrm{B}}_\mathrm{s}$ mixing amplitude $M_{12}$ can be expressed in terms of Wilson coefficients, which contain the short-distance information, and matrix elements of local four-quark operators. The Standard Model contribution involves only a single operator:

$$O_L = \bar{s}_L \gamma_\mu b_L \; \bar{s}_L \gamma^\mu b_L .$$

Its Wilson coefficient $C_L$ is computed from the box diagram involving two W bosons and top quarks. In the CMM model a new operator $O_R$ occurs, which is obtained from $O_L$ by replacing the left-handed fields with right-handed ones. Since $\langle B_s | O_L | \overline{B}_s \rangle = \langle B_s | O_R | \overline{B}_s \rangle$, no new hadronic matrix elements are needed. Using the standard relativistic normalisation $\langle B_s | B_s \rangle = 2EV$ we can write

$$M_{12} = \frac{G_F^2 \, M_W^2}{32\pi^2 \, M_{B_s}} \, \lambda_t^2 \, (C_L + C_R) \, \langle B_s | O_L | \overline{B}_s \rangle . \quad (4)$$

Here $\lambda_t = V_{ts}^* V_{tb}$ comprises the CKM elements. We define all Wilson coefficients and matrix elements at the renomalisation scale $\mu = m_b$, at which the Standard Model coefficient evaluates to $C_L = 8.5$ [5]. The leading contribution to $\mathrm{B_s}-\overline{\mathrm{B}}_\mathrm{s}$ mixing in the CMM model arises from one-loop box diagrams with gluinos and squarks. The result is

$$C_R = \frac{\Lambda_3^2}{\lambda_t^2} \frac{8\pi^2 \alpha_s^2(m_{\tilde{g}})}{G_F^2 M_W^2 m_{\tilde{g}}^2} \left[ \frac{\alpha_s(m_{\tilde{g}})}{\alpha_s(m_b)} \right]^{6/23} S^{(\tilde{g})} . \quad (5)$$

Here

$$|\Lambda_3| = |U_{\mu 3}| |U_{\tau 3}| \approx \frac{1}{2} \quad (6)$$

is the relevant combination of mixing matrix elements in the right-handed sdown sector, and

$$\begin{aligned} S^{(\tilde{g})} = \; & \frac{11}{18} \left[ G(x_{\tilde{b}}, x_{\tilde{b}}) + G(x_{\tilde{s}}, x_{\tilde{s}}) - 2G(x_{\tilde{b}}, x_{\tilde{s}}) \right] \\ & - \frac{2}{9} \left[ F(x_{\tilde{b}}, x_{\tilde{b}}) + F(x_{\tilde{s}}, x_{\tilde{s}}) - 2F(x_{\tilde{b}}, x_{\tilde{s}}) \right] \quad (7) \end{aligned}$$

with $x_{\tilde{q}} = m_{\tilde{q}}^2 / m_{\tilde{g}}^2$ and $\tilde{s}, \tilde{b}$ denoting the right-handed squarks of the second and third generations. The functions $F$ and $G$ are defined in [4]. Note the twofold enhancement of $C_R$ due to the large atmospheric mixing and the large strong coupling constant. This is, however, partially offset by a smaller loop function $S^{(\tilde{g})}$. The neutral $B_s$-meson mass difference is given by

$$\Delta M_{B_s} = 2|M_{12}|. \tag{8}$$

The phase of $M_{12}$ is responsible for mixing-induced CP violation. Note that the phase of $\Lambda_3$ is undetermined, so that there is potentially large, but not predictable, CP violation in decay modes like $B_s \to \psi\phi, \psi\eta^{(\prime)}$. A measurement of $\Delta M_{B_s}$ and the mixing-induced CP asymmetries in one of these decays will allow to determine both magnitude and phase of $M_{12}$ and therefore of $C_R$. If the $B_s$ oscillations are too rapid, these asymmetries cannot be measured. Then still the width difference in the $B_s$ system can be used to determine cos(arg $M_{12}$) [7].

The input parameters for our analysis are $y_t$, $m_{\tilde{g}}$ and the average squark mass $m_{\tilde{q}}$ and the trilinear term $a_d$ of the first generation, all defined at the electroweak scale. They are evolved to the GUT scale and the SU(5) and SO(10) GUT parameters are determined. All other parameters are then determined from the RG flow back down to the electroweak scale. The flavour-changing effects in the CMM model grow with the sizes of the universal soft sfermion mass $m_0$ and the universal trilinear term $a_0$ at the Planck scale. However, a larger $m_0$ also implies larger squark masses leading to a suppression of the gluino box function $S^{(\tilde{g})}$. The effect on $B_s - \overline{B}_s$ mixing is maximal for values of $m_{\tilde{q}}$ around 800 GeV. Furthermore $a_0$ and $m_0$, which for a given gluino mass determine the sfermion mass spectrum, are constrained by the lower bounds on these masses. We have further checked that no charge- or colour-breaking vacuum occurs. Finally the effect decreases with increasing $m_{\tilde{g}}$, so that the maximal effect is found for the experimental lower bound $m_{\tilde{g}} = 195\,\text{GeV}$.

The allowed area in the complex $M_{12}$ plane for the CMM model is depicted in Fig. 2. For simplicity we neglect the 30% uncertainty from current lattice QCD determinations of $\langle B_s | O_L | \overline{B}_s \rangle$. Fixing the value of this matrix elements to the central value quoted in [6] results in a Standard Model prediction of 17.2 $\text{ps}^{-1}$. The small black circle in Fig. 2 indicates this value and the large red disk denotes the range covered by the CMM model. The blue circle centered at the origin is the region excluded by the 95 % CL experimental lower bound [8]

$$\Delta M_{B_s} > 14.4\,\text{ps}^{-1}. \tag{9}$$

We see that in total the mass difference can exceed its Standard-Model prediction by a factor of 16.

In conclusion we have perfomed a RG analysis for the CMM model [1] and computed the impact on $B_s - \overline{B}_s$ mixing. Our work complements and improves previous GUT-inspired analyses (see e.g. [9]), which supplement the MSSM with minimal flavour violation by $\tilde{b}_R$–$\tilde{s}_R$ mixing at the electroweak scale.

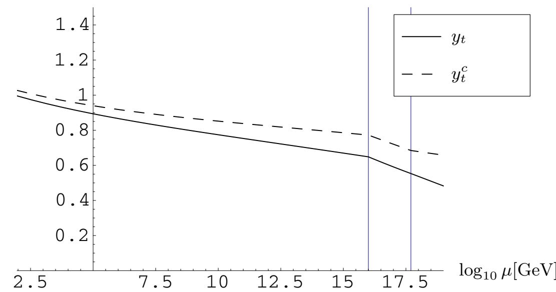


**Fig. 1.** RG evolution of $y_t$. See text for explanation

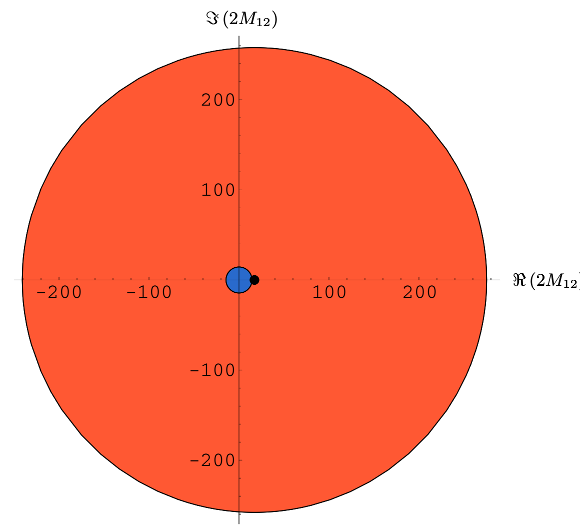


**Fig. 2.** Allowed region in the complex $2M_{12}$ plane. The axis units are inverse picoseconds

## References

1. D. Chang, A. Masiero, and H. Murayama: Phys. Rev. D **67**, 075013 (2003) [arXiv:hep-ph/0205111]
2. N. Cabibbo: Phys. Rev. Lett. **10**, 531 (1963); M. Kobayashi and T. Maskawa: Prog. Theor. Phys. **49**, 652 (1973); B. Pontecorvo: Sov. Phys. JETP **7**, 172 (1958); [Zh. Eksp. Teor. Fiz. **34**, 247 (1957)]; B. Pontecorvo: Sov. Phys. JETP **6**, 429 (1957); [Zh. Eksp. Teor. Fiz. **33**, 549 (1957)]; Z. Maki, M. Nakagawa, and S. Sakata: Prog. Theor. Phys. **28**, 870 (1962)
3. S.P. Martin and M.T. Vaughn: Phys. Rev. D **50**, 2282 (1994) [arXiv:hep-ph/9311340]; Y. Yamada: Phys. Rev. D **50**, 3537 (1994) [arXiv:hep-ph/9401241]
4. S. Bertolini, F. Borzumati, A. Masiero, and G. Ridolfi: Nucl. Phys. B **353**, 591 (1991)
5. A.J. Buras, M. Jamin, and P.H. Weisz: Nucl. Phys. B **347**, 491 (1990)
6. L. Lellouch: Nucl. Phys. Proc. Suppl. **117**, 127 (2003) [arXiv:hep-ph/0211359]
7. Y. Grossman: Phys. Lett. B **380**, 99 (1996) [arXiv:hep-ph/9603244]; I. Dunietz, R. Fleischer, and U. Nierste: Phys. Rev. D **63**, 114015 (2001) [arXiv:hep-ph/0012219]
8. M. Battaglia et al.: arXiv:hep-ph/0304132
9. R. Harnik, D.T. Larson, H. Murayama, and A. Pierce: arXiv:hep-ph/0212180